Graph Theory
Lecture 9: Kruskal's Algorithm

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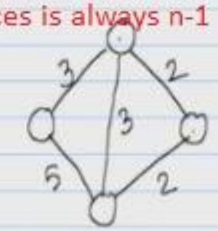
Minimum Spanning Trees
Spanning tree definition:

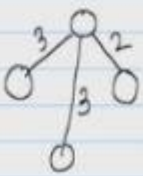
Minimum Spanning Trees (MST)

A spanning tree of a graph G is a subgraph that is a tree which includes all the vertices of G.

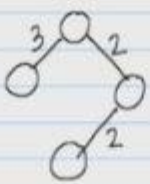
Note: no. of edges in a spanning tree of n vertices is always n-1



Weighted graph G



$Cost(T_1) = 8$

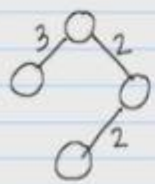


$Cost(T_2) = 7$

Minimum Spanning Tree definition

A minimum spanning tree (MST) of G is a spanning tree of G for which the sum of edge costs is minimum.

From previous slide spanning trees i.e. T_1 and T_2
T_2 is minimum weighted tree



$Cost(T_2) = 7$

T_2 is the unique minimum spanning tree of G

Theorem :

Algorithm : Kruskul's Algorithm for MST

Kruskal's Algorithm

Input: $G = (V, E)$
Output: A MST

$T = \phi$; As initially no node is spanned

$VS = \phi$; VS= Vertex set for spanning tree

for each $v \in V$, do add $\{v\}$ to VS;

order the edges (v, w) in non-decreasing order of weights and store in Q =queue

while ($|VS| > 1$) { |vs|=cardinality or no. of vertices of the graph

choose an edge (v, w) of lowest cost

delete (v, w) from Q

if v and w are in different sets W_1 & W_2 in VS

{ replace W_1 & W_2 in VS by $W_1 \cup W_2$;

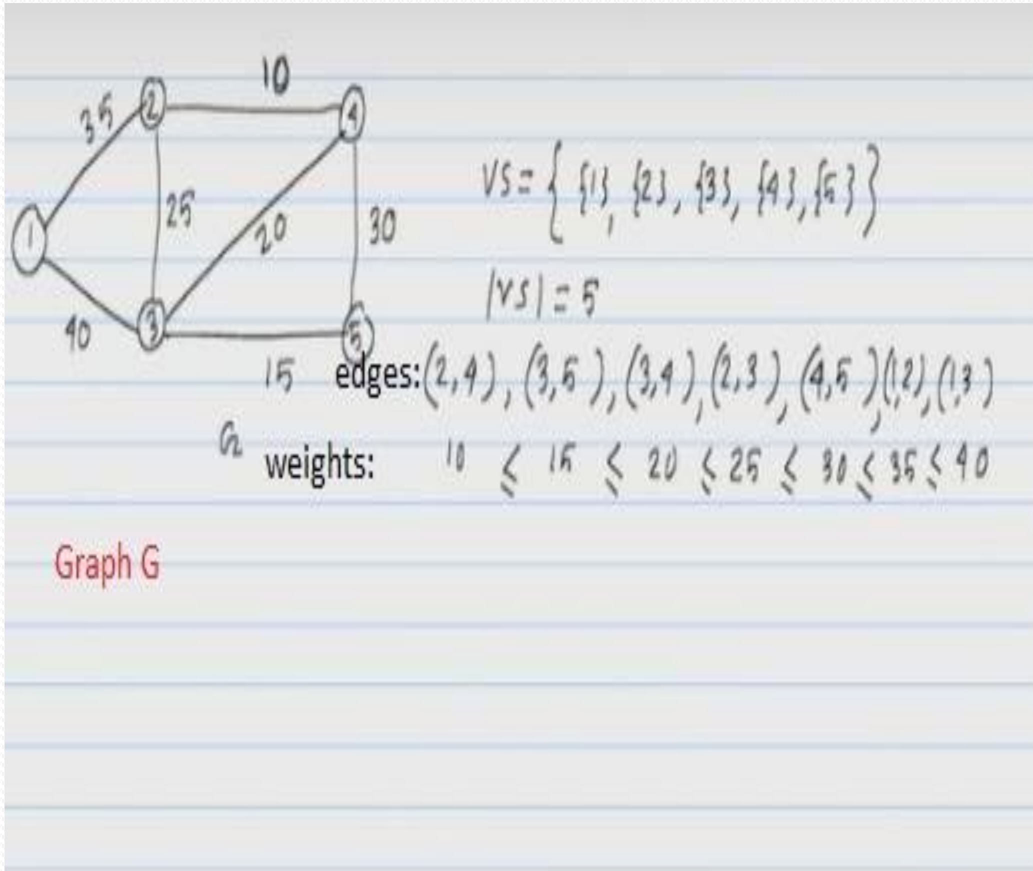
$T = T \cup (v, w)$;

}

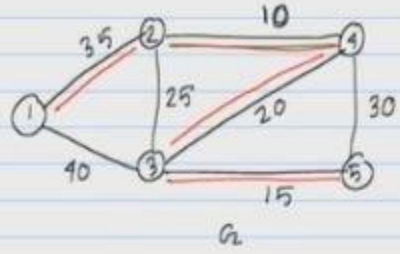
}

return T.

Example: For finding MST Using Kruskal's algorithm



Example(Continues…)



w1 and w2 are components or partitions

$VS = \{ \{1\}, \overset{w_1}{\{2\}}, \{3\}, \overset{w_2}{\{4\}}, \{5\} \}$

$|VS| = 5$

$\cancel{(2,4)}, \cancel{(3,5)}, \cancel{(3,4)}, \cancel{(2,3)}, \cancel{(4,5)}, \cancel{(1,2)}, (1,3)$

$10 \le 15 \le 20 \le 25 \le 30 \le 35 \le 40$

edges	action	VS
(2,4)	add	$\{ \{1\}, \overset{w_1}{\{2,4\}}, \{3\}, \overset{w_2}{\{5\}} \}$
(3,5)	add	$\{ \{1\}, \overset{w_2}{\{2,4\}}, \overset{w_1}{\{3,5\}} \}$
(3,4)	add	$\{ \overset{w_1}{\{1\}}, \overset{w_2}{\{2,3,4,5\}} \}$
(2,3)	reject	
(4,5)	reject	
(1,2)	add	$\{ \{1,2,3,4,5\} \}$

$|VS| = 1$

$T = \{ (2,4), (3,5), (3,4), (1,2) \}$ Then T is MST as is shown above in graph with red

Another Interpretation of Kruskal's algorithm

1. Treat n nodes of the graph as n independent trees of the forest
2. Join first the 2 vertices with minimum possible weights.
3. repeat step 2 until no vertices remains for traversal.